Nonlinear Analysis of Laminated Shells Including Transverse Shear Strains

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Abstract

NUMERICAL results obtained using a doubly curved, shear deformable shell element are presented for geometrically nonlinear anlaysis of laminated composite shells. The element is based on an extension of Sanders' shell theory and accounts for the von Kármán strains and transverse shear strains. The sample numerical results presented here for the geometrically nonlinear analysis of laminated composite shells should serve as references for future investigations.

Contents

Laminated shells are finding increased applications in aerospace, automobile, and petrochemical industries. This is due primarily to the high stiffness-to-weight ratio, high-strength-to-weight ratio, and lower machining and maintenance costs associated with composite structures. However, the analysis of composite structures is more complicated when compared to metallic structures, because laminated composite structures are anisotropic and characterized by bending-stretching coupling. Further, the classical shell theories, which are based on the Kirchhoff-Love kinematic hypothesis, are known to yield deflections and stresses in laminated shells that are as much as 30% in error. This error is due to the neglect of transverse shear strains in the classical shell theories.

The effects of transverse shear deformation and thermal expansion through the thickness for laminated, transversely isotropic, cylindrical shells were considered by Zukas and Vinson.[1] Dong and Tso[2] constructed a laminated orthotropic shell theory that includes transverse shear deformation. Other shear deformation theories, specialized to anisotropic cylindrical shells, were presented by Whitney and Sun.[3,4] Recently, Reddy[5] extended Sanders' theory to account for the transverse shear strains, and presented exact solutions for simply supported cross-ply laminated shells. All of these studies are limited to small displacement theories and static analyses.

In the present paper, the extended Sanders' shell theory that accounts for the shear deformation and the von Kármán strains is used to develop a displacement finite element model for the bending analysis of laminated composite shells. Numerical results for cylindrical and doubly curved shells are presented, showing the effect of geometry and material orthotropy on the deflections and stresses.

Consider a laminated shell constructed of a finite number of uniform-thickness orthotropic layers, oriented arbitrarily with respect to the shell coordinates (ξ_1,ξ_2,ζ). The orthogonal curvilinear coordinate system (ξ_1,ξ_2,ζ) is chosen such that the ξ_1 and ξ_2 curves are lines of curvature on the midsurface $\zeta=0$, and ζ curves are straight lines perpendicular to the surface $\zeta=0$.

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The principle of virtual displacements can be used to derive the governing equations[5,6]:

$$\frac{\partial N_1}{\partial x_1}+\frac{\partial}{\partial x_2}(N_6-c_0M_6)+\frac{Q_1}{R_1}=0$$

$$\frac{\partial}{\partial x_1}(N_6+c_0M_6)+\frac{\partial N_2}{\partial x_2}+\frac{Q_2}{R_2}=0$$

$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial x_2}-\left(\frac{N_1}{R_1}+\frac{N_2}{R_2}-q\right)+\mathfrak{N}(u_3)=0$$

$$\frac{\partial M_1}{\partial x_1}+\frac{\partial M_6}{\partial x_2}-Q_1=0,\quad \frac{\partial M_6}{\partial x_1}+\frac{\partial M_2}{\partial x_2}-Q_2=0 \tag{1}$$

where $c_0=0.5\,(1/R_2-1/R_1)$, and

$$\mathfrak{N}(u_3)=\frac{\partial}{\partial x_1}\left(N_1\frac{\partial u_3}{\partial x_1}+N_6\frac{\partial u_3}{\partial x_2}\right)+\frac{\partial}{\partial x_2}\left(N_6\frac{\partial u_3}{\partial x_1}+N_2\frac{\partial u_3}{\partial x_2}\right) \tag{2}$$

The relationship between the stress resultants (N_i, M_i, Q_i) are generalized displacements $(u, v, w, \phi_1, \phi_2)$ can be found in Ref. 6.

The finite element model of the equations is of the form

$$[K(\Delta)]\{\Delta\}=\{F\} \tag{3}$$

where $\{\Delta\}=\{\{u_1\},\{u_2\},\{u_3\},\{\phi_1\},(\phi_2\}\}^T$, $[K]$ is the element stiffness matrix, and $\{F\}$ is the force vector. The coefficients of the stiffness matrices are included in Ref. 6. Note that the stiffness matrix $[K]$ is a function of the unknown solution vector $\{\Delta\}$; therefore, an iterative solution procedure is required for each load step. In the present study, the authors have used the direct (i.e., Picard-type) iteration technique. Sample numerical results for laminated shell problems are presented in Figs. 1-3. A discussion of these results is presented in the following paragraphs.

The following types of boundary conditions for a quarter-panel were used (along the symmetry lines, normal in-plane displacement, and normal rotation are set to zero):

$$v=w=\phi_1=0 \text{ at } y=b,\quad u=w=\phi_2=0 \text{ at } x=a \tag{4}$$

Here u, v, and w denote the displacements along the three coordinate axes, and ϕ_1 and ϕ_2 denote the rotations of the transverse normals about the x_2 and x_1 axes, respectively. The following material properties are used:

Material 1:

$$E_1=25\times10^6 \text{ psi}, E_2=10^6 \text{ psi}, G_{12}=G_{13}=0.5\times10^6 \text{ psi}$$
$$G_{23}=0.2\times10^6 \text{ psi}, \nu_{12}=0.25 \tag{5a}$$

Material 2:

$$E_1=40\times10^6 \text{ psi}, E_2=10^6 \text{ psi}, G_{12}=G_{13}=0.6\times10^6 \text{ psi}$$
$$G_{23}=0.5\times10^6 \text{ psi}, \nu_{12}=0.25 \tag{5b}$$

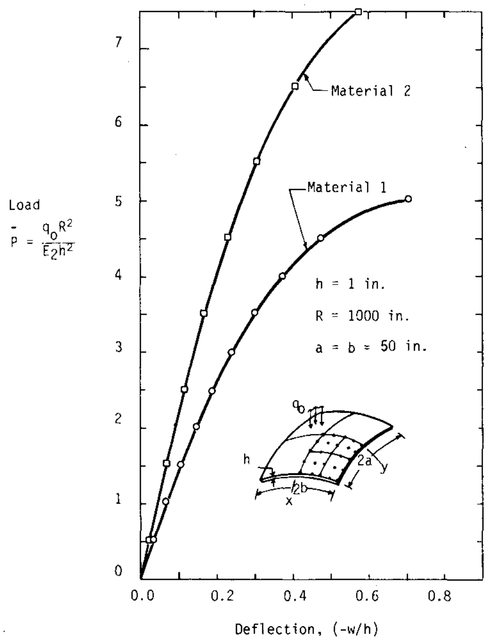

Fig. 1 Bending of a nine-layer cross-ply [0°/90°/0°/...] simply supported spherical shell subjected to a uniformly distributed load.

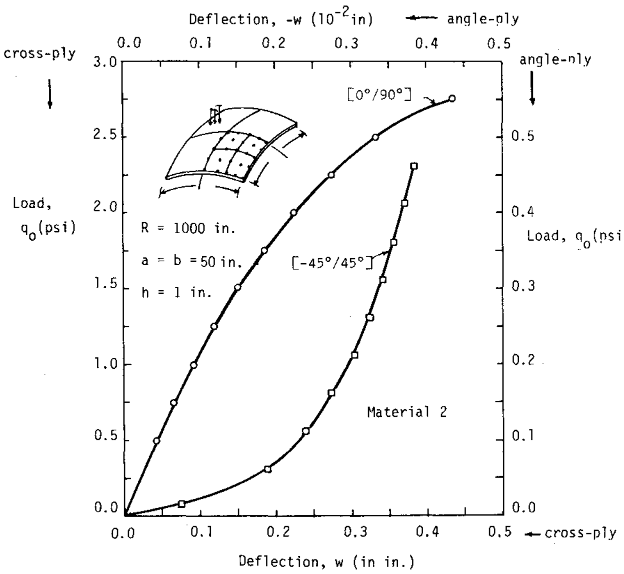

Fig. 2 Bending of a two-layer cross-ply and angle-ply simply supported spherical shells under uniform loading.

The first problem is concerned with the bending of a nine-layer [0°/90°/0°/90°/...] cross-ply spherical panel under external pressure load. The following geometrical data are used in the analysis: $R_1 = R_2 = R = 1000$ in., $a = b = 50$ in., $h = 1$ in. Individual layers are assumed to be of equal thickness ($h_i = h/9$). Results using two sets of orthotropic-material constants, typical of high-modulus graphite-epoxy material (the ratios are more pertinent here) for individual layers, are presented in Fig. 1.

Figure 2 contains the pertinent data and results (with different scales) for two-layer cross-ply and angle-ply simply supported spherical panels (of material 2). It is interesting to note that the type of nonlinearity exhibited by the two shells is quite different; the cross-ply shell gets softer, whereas the angle-ply shell gets stiffer with an increase in the applied load. While both shells have bending-stretching coupling due to the lamination scheme ($B_{22} = -B_{11}$ nonzero for the cross-ply shell and B_{16} and B_{26} are nonzero for the angle-ply shell), the angle-ply experiences shear coupling that stiffens the spherical shell relatively more than the normal coupling (note that, in general, shells get softer under externally applied pressure loads).

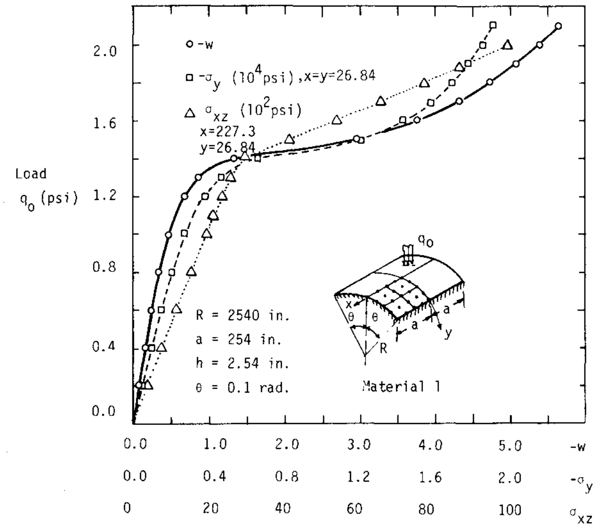

Fig. 3 Bending of a clamped cross-ply [0°/90°] cylindrical shell under a uniform loading.

Lastly, Fig. 3 contains results (i.e., w, σ_y, σ_{xz} vs load) for a cross-ply [0°/90°] clamped cylindrical panel under uniform load. Clearly, the load deflection curve indicates that the nonlinearity exhibited by the cross-ply shell is of the hardening type.

The shear-flexible finite element presented here is computationally more efficient than those based on fully or degenerated three-dimensional elasticity theory. Since only the von Kármán-type nonlinearity is accounted for here, the element is not suitable for applications involving severe distortions (i.e., large strains). From the numerical computations it is observed that boundary conditions on the inplane displacements have a significant effect on the shell deflections and stresses. It is also observed that the form of nonlinearities exhibited by laminated shells depends on the lamination schemes (i.e., cross-ply and angle-ply). It would be of considerable interest to verify these findings by experiments.

Acknowledgments

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